

Differentiation and Integration Rules

This handout highlights some of the most frequently encountered rules for differentiation and integration.

For the following, let u and v be functions of x . Let c be a constant.

Let a , c and C be constants.

Fundamental Rules

$$\frac{d(c)}{dx} = 0 \qquad \int c \, dx = cx + C$$

$$\frac{d(cu)}{dx} = c \frac{du}{dx} \qquad \int c \, du = c \int du + C$$

$$\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \int (du + dv) = \int du + \int dv + C$$

$$\frac{d(u^n)}{dx} = nu^{n-1} \frac{du}{dx} \qquad \int u^n \, du = \frac{u^{n+1}}{n+1} + C$$

$$\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx} \qquad \int u \, dv = uv - \int v \, du + C$$

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Trigonometric Functions

$$\frac{d(\sin u)}{dx} = \cos u \frac{du}{dx} \qquad \int \sin u \, du = -\cos u + C$$

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$$\frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx} \qquad \int \tan u \, du = \ln |\sec u| + C$$

$$\frac{d(\cot u)}{dx} = -\csc^2 u \frac{du}{dx} \qquad \int \cot u \, du = \ln |\sin u| + C$$

$$\frac{d(\sec u)}{dx} = \sec u \tan u \frac{du}{dx} \qquad \int \sec u \, du = \ln |\sec u + \tan u| + C$$

Trigonometric Functions Continued

$$\frac{d(\csc u)}{dx} = -\csc u \cot u \frac{du}{dx} \quad \int \csc u \, du = -\ln |\csc u + \cot u| + C$$

$$\int \sec^2 u \, du = \tan u + C \quad \int \csc^2 u \, du = -\cot u + C$$

Exponential and Logarithmic Functions

$$\frac{d(e^u)}{dx} = e^u \frac{du}{dx} \quad \int e^u \, du = e^u + C$$

$$\frac{d(\ln u)}{dx} = \frac{1}{u} \frac{du}{dx} \quad \int \frac{1}{u} \, du = \ln |u| + C$$

$$\frac{d(a^u)}{dx} = (a^u)(\ln a) \frac{du}{dx} \quad \int a^u \, du = \frac{a^u}{\ln a} + C$$

$$\frac{d(u^v)}{dx} = u^v(v' \ln u + \frac{v}{u} u')$$

Inverse Trigonometric Functions

$$\frac{d(\sin^{-1} u)}{dx} = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \quad \int \frac{1}{\sqrt{a^2-u^2}} = \sin^{-1} \frac{u}{a} + C$$

$$\frac{d(\cos^{-1} u)}{dx} = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx} \quad \int \frac{-1}{\sqrt{a^2-u^2}} = -\sin^{-1} \frac{u}{a} + C$$

$$\frac{d(\tan^{-1} u)}{dx} = \frac{1}{1+u^2} \frac{du}{dx} \quad \int \frac{1}{a^2+u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\frac{d(\cot^{-1} u)}{dx} = \frac{-1}{1+u^2} \frac{du}{dx} \quad \int \frac{-1}{a^2+u^2} = -\frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\frac{d(\sec^{-1} u)}{dx} = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx} \quad \int \frac{1}{|u|\sqrt{u^2-a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right| + C$$

$$\frac{d(\csc^{-1} u)}{dx} = \frac{-1}{|u|\sqrt{u^2-1}} \frac{du}{dx} \quad \int \frac{-1}{|u|\sqrt{u^2-a^2}} = -\frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right| + C$$

Hyperbolic Trigonometric Functions

$$\frac{d(\sinh u)}{dx} = \cosh u \frac{du}{dx} \quad \int \sinh u \, du = \cosh u + C$$

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$$\frac{d(\tanh u)}{dx} = \operatorname{sech}^2 u \frac{du}{dx} \quad \int \tanh u \, du = -\ln |\operatorname{sech} u| + C$$

Inverse Hyperbolic Trigonometric Functions

$$\frac{d(\sinh^{-1} u)}{dx} = \frac{1}{\sqrt{1+u^2}} \frac{du}{dx} \quad \int \frac{1}{\sqrt{1+u^2}} \, du = \sinh^{-1} u + C$$

$$\frac{d(\cosh^{-1} u)}{dx} = \frac{1}{\sqrt{u^2-1}} \frac{du}{dx} \quad \int \frac{1}{\sqrt{u^2-1}} \, du = \cosh^{-1} u + C$$

$$\frac{d(\tanh^{-1} u)}{dx} = \frac{1}{1-u^2} \frac{du}{dx} \quad \int \frac{1}{1-u^2} \, du = \tanh^{-1} u + C$$